ANSWER Tutorial 2: Languages

1. Consider the language $L$ of all strings of $a$'s and $b$'s that do not end with $b$ and do not contain the substring $bb$. Find a finite language $S$ such that $L = S^*$. $L = \{a, \text{ba}\}^*$

2. i) Give a regular expression describing the set of all strings over $\{a, b, c\}$ in which all $a$'s precede the $b$'s, which in turn precede the $c$'s. It is possible that there are no $a$'s, $b$'s, or $c$'s. $a^*b^*c^*$

   ii) The same as the previous, but without the empty string. $a^*b^*c^* + a^*b^*c^* + a^*b^*c^*$

3. Give a regular expression describing the set of all strings over $S = \{a, b\}$ in which the string contain at least two $b$'s. $bb^* + a*bb*a^* + a*ba*b^*b^*a^*$

4. Let $X = \{a\}$ and $Y = \{b\}$. For $n = 0, 1, 2, 3$, list the strings:
   a. $X^nY = Y = \{b\}$
   $X^1Y = \{ab\}$
   $X^2Y = \{aab\}$
   $X^3Y = \{aaab\}$
   b. $XY^n$
   $XY^0 = \{a\}$
   $XY^1 = \{ab\}$
   $XY^2 = \{abb\}$
   $XY^3 = \{abbb\}$
   c. $(XY)^n$
   $(XY)^0 = \{\lambda\}$
   $(XY)^1 = \{ab\}$
   $(XY)^2 = \{abab\}$
   $(XY)^3 = \{ababab\}$

5. Generate possible strings for each of the following regular expression recursively (at least for 4 iteration):
   a. $(a + b)^*$
      $(a + b)^0 = \lambda$
      $(a + b)^1 = (a + b) = a \ b$
      $(a + b)^2 = (a + b) (a + b) = aa \ aba \ bba$
      $(a + b)^3 = (a + b) (a + b) (a + b) = aaa \ aab \ aba \ baa \ bab \ bba \ bbb$
      $(a + b)^4 = (a + b) (a + b) (a + b) (a + b) = aaaa \ aabab \ ababa \ ababa \ baaaa \ baababa \ baaababa \ bbbaba \ bbababa \ bbbbaba \ bbbbba$
      So $\lambda, a b a a b b a a a b b b a b b b a b b b a b b b b a b b b a b b b a b b b a b b b a b b b a b b b ...$

   b. $(a \cup b)^*ba$
      $(a \cup b)^0ba = \lambda ba$
      $(a \cup b)^1ba = (a \cup b) ba = aba bba$
\[(a \cup b)^2 ba = (a \cup b)(a \cup b) ba = aaba abba baba bbba \]
\[(a \cup b)^3 ba = (a \cup b)(a \cup b) (a \cup b) ba = aaaba aababa ababa bababa bababa bbaba bbbbba \]
\[(a \cup b)^4 ba = (a \cup b)(a \cup b)(a \cup b) (a \cup b) ba = aaaaba aaabba aababa aaabba ababa ababba abbaba bbbba \]
\[= ba aba bba aaaba aabba ababa abbbba baaba aabba baababa bbbba bbaaba babbba bbbaba bbbaba bbbaba... \]

c. \[(a + ab + aab)^* = (a + ab + aab)^0 = \lambda \]
\[(a + ab + aab)^1 = (a + ab + aab) = a ab aab \]
\[(a + ab + aab)^2 = (a + ab + aab)(a + ab + aab) = aa aab aaab aba abab abbaab \]
\[(a + ab + aab)^3 = (a + ab + aab)(a + ab + aab)(a + ab + aab) = aaa aaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab aabaab \]

6. Generate 3 strings for each of the following regular expression:
   a. \[(a+b)^*ba = ba aba bba abba \]
   b. \[(a + ab + aab)^* = a ab aab abaab aab \]
   c. \[(ab)^* + a^* = ab a ba aab abab \]
   d. \[(ba+bb)^* + (ab+aa)^* = ba bb ab aa babb abaa \]
   e. \[(a+b)^*ab(a+b)^* = ab aab bab aba abb baba abba \]

7. Give a description of the following languages in your own words.
   a. \[(a+b)^*ba \] all strings over \{a, b\} that end with \(ba\)
   b. \[aa(a + b)^*aa \] all strings over \{a, b\} that start and end with \(aa\)

8. Give regular expression for the following languages over the alphabet \(\Sigma = \{a, b\}\).
   (i) All strings except empty string. \((a + b)^*\)
   (ii) All strings in which \(a\), if it exists, appears tripled. This means that every clump of \(a\)'s contains 3 \(a\)'s, or 6 \(a\)'s, or 9 \(a\)'s, and so on. \((aaa)+b)^*\)
   (iii) All strings that starts with even number of \(a\)'s; followed by odd number of \(b\)'s; and end with even number of \(a\)'s. \((aa)^*b'(bb)^*(aa)^*\)
   (iv) All strings that starts with \(a\) and has odd length; OR all strings that starts with \(b\) and has even length. \(a(aa)^* + bb(bb)^*\)

9. Give a regular expression for the following language \(L\).
   a. \(L = \) all strings over \{a,b\} that begin and end with \(aa\) \(aa(a + b)^*+(a + b)^*aa\)
   b. \(L = \) all strings over \{a,b\} that contain the substring \(aa\) or \(bb\) \((aa)^* + (bb)^*\)
ANSWER Tutorial 3: CFG

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| $S \to aaSB \mid \lambda$  
$B \to bB \mid b$ | $L(G) = (V, \Sigma, P, S)$  
Where: $V = \{S, B\}$  
$\Sigma = \{a, b\}$  
$P: S \to aaSB \mid \lambda$  
$B \to bB \mid b$ | $aab$ | $aaaabbb, aaaaaabbb$ |
| $S \to aSb \mid A$  
$A \to cAd \mid cBd$  
$B \to aBb \mid ab$ | $L(G) = (\{S, A, B\}, \{a, b, c, d\}, P, S)$  
Where $P: S \to aSb \mid A$  
$A \to cAd \mid cBd$  
$B \to aBb \mid ab$ | $cabd$ | $acabdb, aacccabdddb, acaabbbdb, aacaabbbdbb$ |

2. Let $L$ be the language over $\{a, b\}$ consisting of all strings with at least one $aa$ in them somewhere.
   a) Regular expression defining $L$: $(a+b)^*(aa(a+b)^*)$
   b) Design a regular grammar for $L$.  
   $S \to AaaA$  
   $A \to aA \mid bA \mid \lambda$

3. Let $G$ be the grammar:
   $S \to SAB \mid \lambda$  
   $A \to aA \mid a$  
   $B \to bB \mid \lambda$
   a) Give a leftmost derivation of $aabaaabb$.
      $S \Rightarrow SAB \Rightarrow SABAB \Rightarrow aBAB \Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB$
      $\Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB$
      $\Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB$
      $aBABB \Rightarrow aBABB \Rightarrow aBABB \Rightarrow aBABB$
   b) Draw the derivation tree:

   ![Derivation Tree Image]

   c) Give a regular expression for $L(G)$. $(a*ab^+)^*$

4. Write a CFG for the language $(0 + 1)^*111$.  
   $S \to U111$  
   $U \to 0U \mid 1U \mid \lambda$

5. For each of the following regular expression, design the regular grammar.
a) \( b^*ab^*ab^*a \)
   
   *Analysis:* the language will produce strings with exactly 3 a’s and end with a. In between of the a’s can have as many b’s as we like. The pattern of the string will be: \( BaBaBa. \)
   
   \[
   \begin{align*}
   S & \rightarrow bS \mid aA \\
   A & \rightarrow bA \mid aB \\
   B & \rightarrow bB \mid a
   \end{align*}
   \]

b) \( a^*b(bb)^*aa^* \)
   
   *Analysis:* the language will produce strings with at least one a and one b. The pattern of the string will be: \( AbBBaA. \)
   
   \[
   \begin{align*}
   S & \rightarrow aS \mid bA \\
   A & \rightarrow bbA \mid aB \\
   B & \rightarrow aB \mid \lambda
   \end{align*}
   \]

6. For each of the following regular grammar, give a regular expression for the language generated by the grammar.

   a) \( S \rightarrow aA \\
       A \rightarrow aA \mid bA \mid b \)
       
       *Regular exp:* \( a(a+b)^*b \)

   b) \( S \rightarrow aS \mid bA \\
       A \rightarrow bB \\
       B \rightarrow aB \mid \lambda \)
       
       *Regular exp:* \( a^*bba^* \)